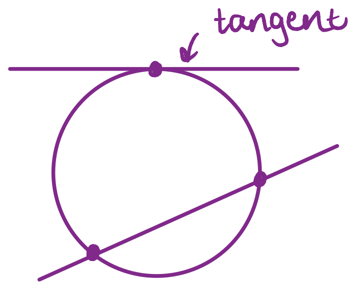
one point of intersection

1. Prove algebraically that the straight line with equation $x - 2y = 10$ is a tangent to the circle with equation $x^2 + y^2 = 20$



$x - 2y = 10$

$(+2y) \quad (+2y)$

$x = 2y + 10$

$x^2 + y^2 = 20$

$(2y+10)^2 + y^2 = 20$

$4y^2 + 20y + 20y + 100 + y^2 = 20$

$5y^2 + 40y + 100 = 20$

$(-20) \quad (-20)$

$5y^2 + 40y + 80 = 0$

$(\div 5) \quad (\div 5)$

$y^2 + 8y + 16 = 0$

$(y+4)(y+4) = 0$

$(y+4)^2 = 0$

$y = -4$

Therefore, the straight line is a tangent to the circle, because there is only one point of intersection

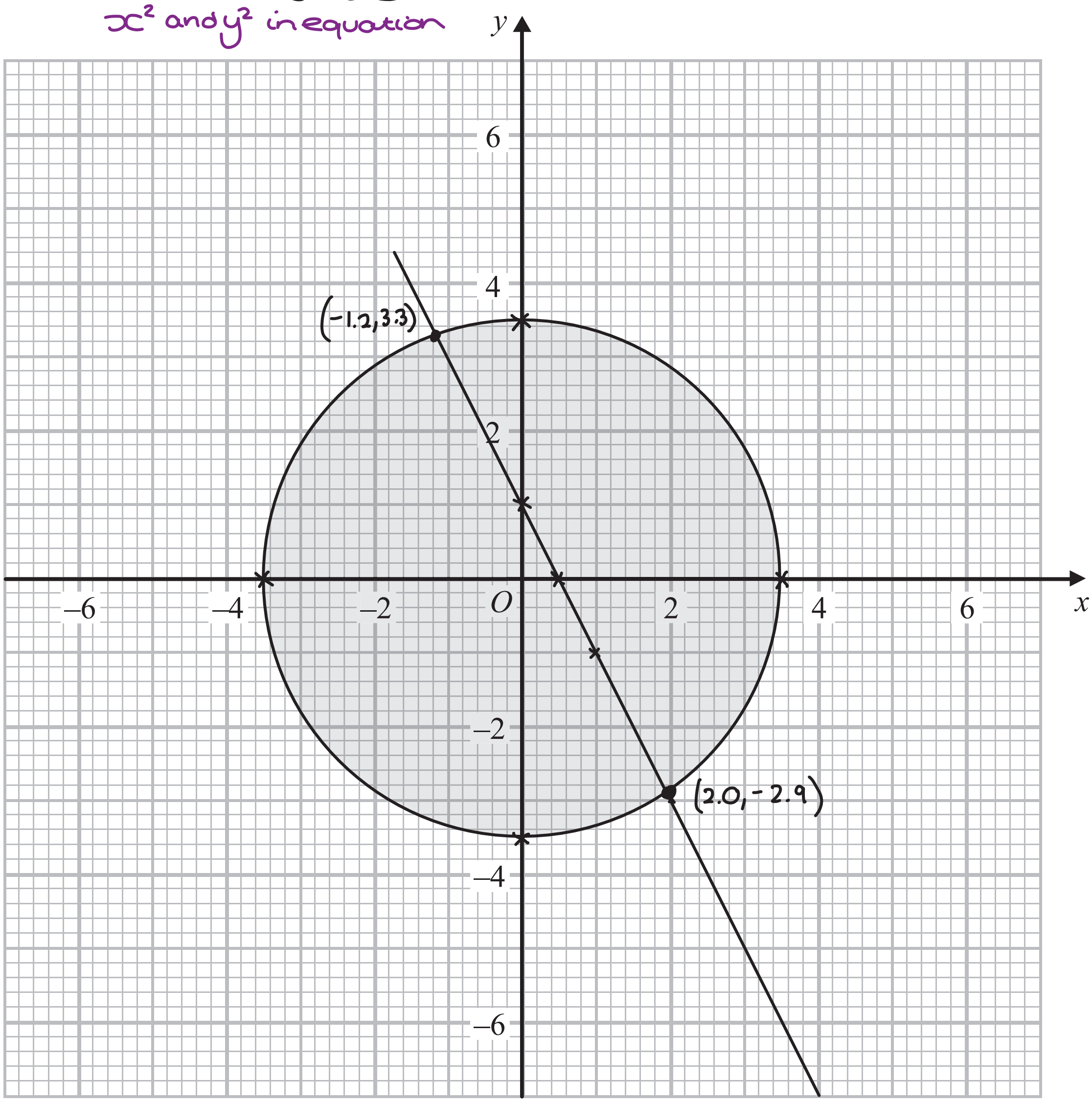
(Total for Question is 5 marks)

centre (0,0)

2. (a) On the grid, draw the graph of $x^2 + y^2 = 12.25$ radius = $\sqrt{12.25} = 3.5$

Circle

x^2 and y^2 in equation



(2)

(b) Hence find estimates for the solutions of the simultaneous equations

$$x^2 + y^2 = 12.25$$
$$2x + y = 1$$

1 Draw on grid - find where it meets the circle

2 correct

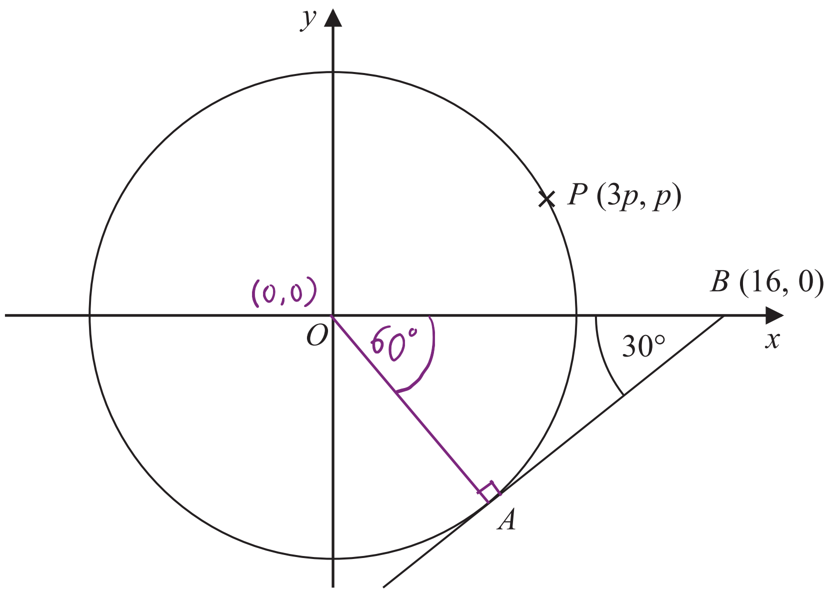
1 1

$x = 2.0$ $y = -2.9$, $x = -1.2$ $y = 3.3$

(3)

(Total for Question is 5 marks)

3. The diagram shows a circle, centre O.

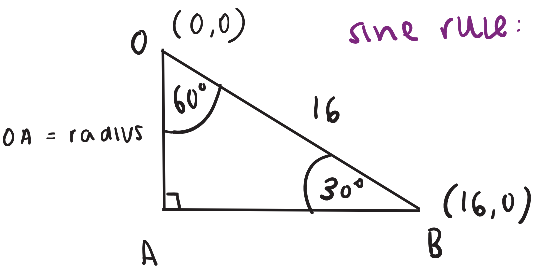


AB is the tangent to the circle at the point A.
Angle $OBA = 30°$

Point B has coordinates (16, 0)
Point P has coordinates $(3p, p)$

Angle between tangent and radius = 90°

Find the value of p.
Give your answer correct to 1 decimal place.
You must show all your working.



sine rule: $\frac{\sin 90}{16} = \frac{\sin 30}{OA}$ (1)

$\frac{1}{16} = \frac{1/2}{OA}$. $OA = 16 \times \frac{1}{2} = 8$.

(1)

$x^2 + y^2 = r^2 \longrightarrow r = 8 \therefore x^2 + y^2 = 8^2$

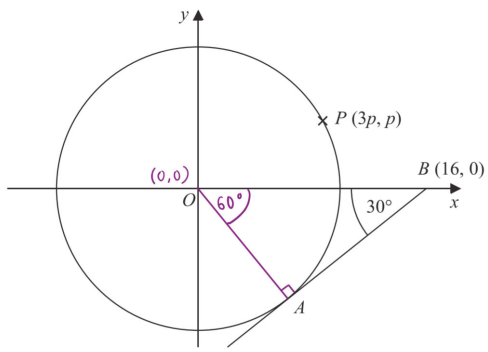
Point $P(3p, p) \longrightarrow x = 3p,\ y = p$.

$\therefore (3p)^2 + p^2 = 8^2$ (1)

$9p^2 + p^2 = 8^2$

$10p^2 = 64$

$p^2 = \frac{64}{10} \quad \therefore p = \sqrt{\frac{64}{10}} = \underline{\underline{2.5}}$ (1 d.p.)



(1)

$p =$ ....2.5........

(Total for Question is 4 marks)

4. **C** is a circle with centre the origin.

A tangent to **C** passes through the points (–20, 0) and (0, 10)

Work out an equation of **C**.
You must show all your working.

$m_T = \frac{\Delta y}{\Delta x} = \frac{10-0}{0--20} = \frac{1}{2}$

$m_r \times m_T = -1$ (Since tangent and radius are perpendicular, product of gradients is -1)

$m_r \times \frac{1}{2} = -1$ $\times 2$

$m_r = -2$

finding equation of tangent : $y = \frac{1}{2}x + c \rightarrow (0,10) \rightarrow y = \frac{1}{2}x + 10$

finding equation of radius : $y = -2x + c \rightarrow (0,0) \rightarrow y = -2x$

(radius goes through origin, (0,0))

find point of intersection : $\frac{1}{2}x + 10 = -2x$

$\frac{5}{2}x = -10 \rightarrow x = -4$

$y = -2x = (-2)(-4) = 8$

↳ $(-4, 8)$

Radius $= \sqrt{(-4)^2 + 8^2}$

$= \sqrt{16 + 64} = \sqrt{80}$

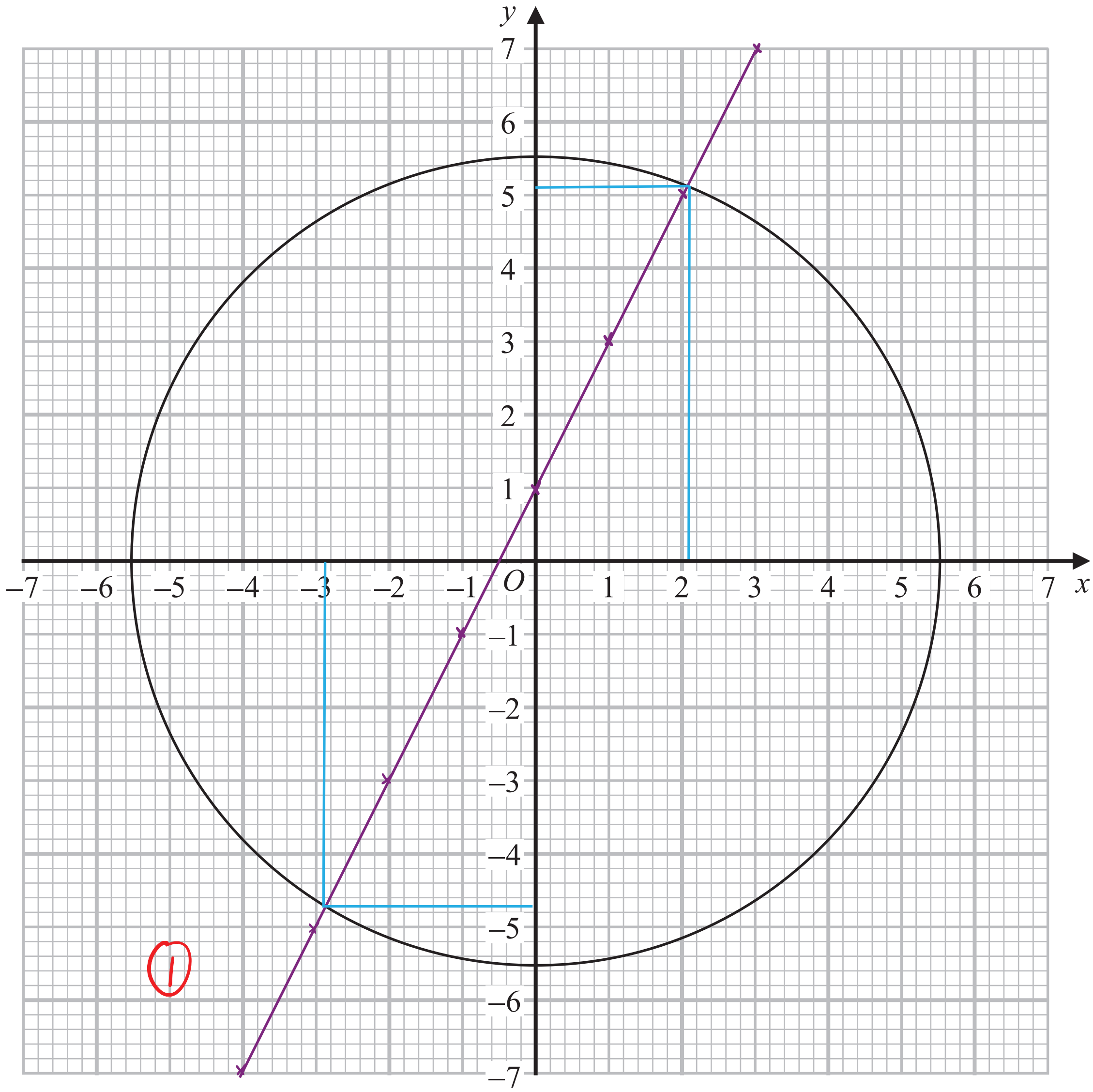
eqn of circle : $x^2 + y^2 = r^2$

$x^2 + y^2 = 80$

....

(Total for Question is 5 marks)

5. The diagram shows the graph of $x^2 + y^2 = 30.25$



Use the graph to find estimates for the solutions of the simultaneous equations

Point at which the circle and line intersect.

$x^2 + y^2 = 30.25$
$y - 2x = 1$ $\therefore$ $y = 2x + 1$.

Gradient = $\frac{\text{change in } y}{\text{change in } x}$ = 2.

1 1

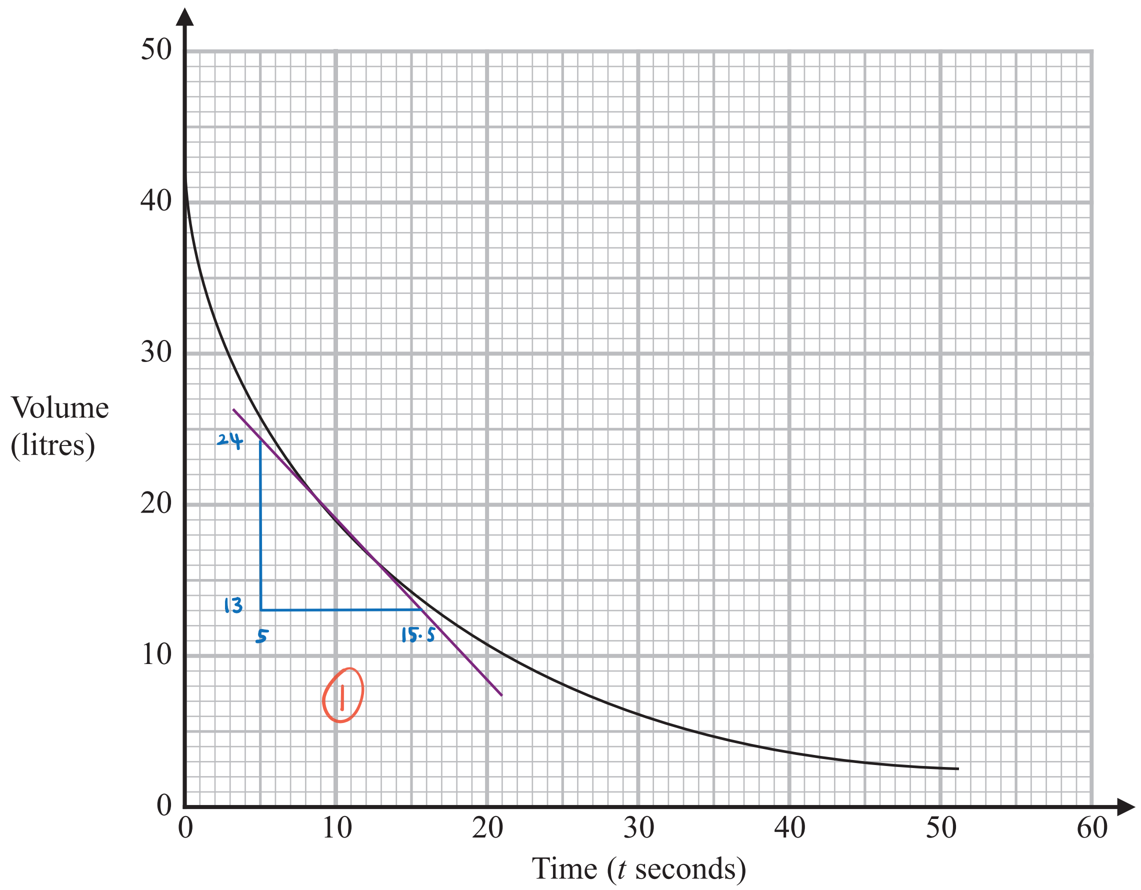
$2 = \frac{2}{1} \therefore \Delta y = 2$
$\Delta x = 1$.

Δ is the Greek letter Delta, which, in Maths, generally means 'change in'.

(2.1, 5.1) and (−2.9, −4.7).

(Total for Question is 3 marks)

6. The graph gives the volume of water, in litres, in a container at time t seconds after the water started to flow out of the container.



Using the graph, work out an estimate for the rate at which the water is flowing out of the container when $t = 12$

You must show your working.

gradient!

When we need to find the gradient of a curve, we have to draw a tangent at a specific point.

↳ in our case, this is at $t = 12$.

Gradient of tangent $= \dfrac{\text{change in } y}{\text{change in } x}$

1

$= \dfrac{24-13}{15.5-5} = 1.047\ldots \approx \underline{\underline{1.0}}$

1

1.0 .... litres per second

(Total for Question is 3 marks)